Individual tree growth and yield models for red oak - sweetgum stands on Mid-South minor stream bottoms producing volume by log grade

By

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Bottomland hardwood stands of the Mid-South region of the United States are some of the most productive forests in the country. A large percentage of these stands are owned by nonindustrial private forest landowners, who have little information on which to base management decisions. These stands are, therefore, a largely unmanaged and under-utilized reserve of high quality hardwoods. To provide landowners with a decision-making tool for comparing management scenarios, a growth and yield study was initiated in 1981. One hundred and fifty permanent plots were installed in red oak-sweetgum stands. The study has been remeasured three times over the past 35 years. New plots were added when losses occurred due to natural disasters or harvesting. Stand level (Iles 2008), log grade volume distribution (Banzhaf 2009), and diameter distribution (Howard 2011) models were developed as component models of the overall growth and yield system. This study completes the modeling effort by developing individual tree equations for percent annual diameter growth and survival. Equations were constructed using linear, non-linear, and logistic regression techniques. The best set of developed equations was selected based on biological consistency, joint behavior when inserted into
the growth and yield computer model, and the performance of each plot’s predicted future yield when compared to its observed data at the next projection period. Final independent stand level variables for the two models included age, diameter at breast height, trees per acre, and average height of dominant trees. Percent diameter growth and survival equations exhibited high fit statistics and when coupled with the other equations in the computer model, produced estimates for trees per acre, basal area, arithmetic and quadratic mean diameters with low bias and root mean squared error. The resulting growth and yield simulator implemented in Microsoft Visual Basic® Editor within Microsoft Excel® enables forest professionals and landowners to make better management decisions for their red oak-sweetgum mixture bottomland hardwood stands by projecting current forest inventories into the future, predicting average yields, and evaluating and comparing forest management scenarios.
DEDICATION

To my Lord and Savior Jesus Christ: thank you for continuing to open doors for me, blessing me with a wonderful family, the wisdom and strength to see things through, and all Your many blessings which are too numerous to count.

To Christy, thank you for always supporting me and never letting me give up. This has taken several years to accomplish, because we “leap frogged” through graduate school. It’s now your turn.

To my children, thanks for keeping Daddy grounded and focused on the goal of providing a better life for you. I would also like to thank you for all the hugs and kisses throughout the years. Please don’t grow up too fast.

To my parents, Larry and Beth Jeffreys, thank you for providing me with support, guidance, and a loving home.
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CHAPTER I
INTRODUCTION

Bottomland hardwoods of the Southern United States are some of the most diverse and productive ecosystems in North America (Banzhaf 2009). They contribute to soil stabilization, water quality, wildlife species diversity, and with proper management produce high value quality timber for a number of products. One of the more important forest types in this area is the red oak \textit{[Quercus Labatae]}-sweetgum \textit{[Liquidambar styraciflua]} mix (Schultz \textit{et al.} 2010). These forests provide material for the production of high quality furniture and flooring, along with veneer for many other products; however, their management or the lack there of has been problematic for private landowners. Landowners and forest managers are reluctant to make management decisions due to a lack of knowledge (Measells \textit{et al.} 2005) about hardwoods and the absence of decision-making models. Without scientifically based knowledge, landowners can only speculate on the outcome of hardwood management decisions.

Growth and yield model systems are designed to assist landowners and forest managers in decision-making by allowing comparisons of management regimes (Rauscher \textit{et al.} 2000, Iles 2008). A growth and yield model that describes log volumes, grade, and stand development is vital to the effective management and sustainability of these red oak-sweetgum forests (Schultz \textit{et al.} 2010).
Hardwood growth and yield modeling research is limited (Rauscher et al. 2000), and the absence of research is especially true for minor stream bottoms in the Southern United States (Perkins et al. 1994). In the past, growth and yield research has primarily focused on single species such as loblolly pine (*Pinus taeda*) (Schultz et al. 2010), due to their commercial value in the pulp and paper, and construction lumber industries. Mixed stands with uneven-aged management are more complex and difficult to model due to the different growth habits of the diversity of species present and varying stand densities. Mixed species growth and yield models are equally important to landowners and managers because of the high value and quality of timber they often produce as in the case with red oak-sweetgum forests.

The need for a red oak-sweetgum forest growth and yield production system by forest professionals and landowners was the basis for a study designed to

1. project current forest inventories into the future,

2. predict average yields from a bare ground scenario, and

3. evaluate and compare forest management scenarios.

Specific system objectives of this study were to

1. create individual tree growth component models for diameter growth and survival,

2. reconcile individual tree models with observed stand level and tree models, and

3. implement models in a Microsoft (MS) Excel Visual Basic (VB) Editor® program incorporating tree grade volume predictions (Banzhaf 2009) and make the program available over the World Wide Web for forestry extension applications.
CHAPTER II

LITERATURE REVIEW

All forest stands are different and, therefore, not all growth and yield models are appropriate for every stand. For example, a model developed for an even-aged pine stand would not be adequate for an uneven-aged oak stand. Models are also different for stands that have varying densities, different diameter distributions, and multiple species, among other variables.

Davis et al. (2005) reviewed three general types of growth and yield models: whole stand models that make predictions for the entire timber stand as a whole by age or basal area per acre; diameter class models that are based on the average tree in each diameter class; and individual tree models that typically give the best simulation of how different forest management scenarios affect tree growth. These three different model types can further be broken down into subcategories. The whole stand model can either be a density-free model or a variable-density model, depending on whether or not a measure of stand density is an independent variable (Davis et al. 2005). Diameter distribution models can be either direct parameter prediction or moment recovery (Avery and Burkhart 2002).

The individual tree model is the most flexible of the three groups because it individually models each tree on a sample tree list (Davis et al. 2005). Individual tree
models are broken down into distance-dependent and distance-independent models. Distance-dependent models use each tree’s distance from surrounding trees to calculate crown competition which is, in turn, used to determine the probability of the tree’s survival and DBH growth. Distance-independent models only use tree and stand characteristics to predict crown competition. Even though each timber stand and tree are unique, growth and yield models are developed to be as robust as possible by including the most important factors and relationships influencing localized variation. However, sufficient differences in growth habits and conditions exist between physiographic regions and stand origins to necessitate many separate models.

**Single Species Models**

Single species growth and yield models for highly valued and easily managed species, such as loblolly [*Pinus taeda*], slash pine [*Pinus elliottii*] (Baldwin and Cao 1999), and longleaf pine [*Pinus palustris*] (Farrar and Matney, 1994) have been the primary focus of past research. Motivation for this research was commercial value, ease of plantation management, and distribution of species. Compared to stands of mixed-species, plantations cover the least amount of area in the southeastern U.S. but their establishment and management have received a large amount of attention and research effort (Baldwin and Cao 1999). Rising demand for forest products in the last several decades together with the conversion of natural forest stands to plantations (Baldwin and Cao 1999) warranted the development of single species growth and yield models. In addition to natural versus plantation scenarios, specific models have been created for cutover and old field stand origins, and thinned and unthinned stands plus other silvicultural treatments.
Lenhart (1972) developed an individual tree diameter distribution growth and yield model for unthinned plantation loblolly pine on old field sites in the interior west Gulf Coastal Plain. The beta distribution was utilized to model diameter distribution and log of height versus reciprocal DBH to model individual tree height. Graphical trends of differences in expected versus observed yields over age, height of dominants, and trees per acre revealed no bias.

Matney and Sullivan (1982) were the first to produce compatible diameter distribution growth and yield equations. These equations were developed for determining stand and stock tables for thinned and unthinned old field pine plantations from data collected in Arkansas, Mississippi, and Tennessee. Compatible growth projection equations were developed from initial stand conditions per acre values of number of trees, basal area, and total tree cubic foot volume. Equations derived showed no evident prediction biases between: 1) observed and predicted average annual mortality and 2) volume and basal area growth rates within treatments across site indices and initial stand parameters. A strong agreement was reported between the observed and predicted initial and projected stand tables. Matney and Sullivan (1982) stated that their prediction equations could be valuable in predicting stand structure in both thinned and unthinned stands for both loblolly pine as well as other pine species.

Baldwin and Feduccia (1987) used the three parameter Weibull distribution to develop growth and yield equations for thinned and unthinned loblolly pine plantations on cutover sites of the West Gulf Coast region of the U.S. Their equations predicted cubic- and board-foot volume, green-weight, and dry-weight yields per unit area of wood only or wood with bark of entire tree boles, boles to any top diameter limit, and branches.
Predictions were tested against the data used to produce these equations and were within plus or minus 5% of observed values. This indicated that their entire prediction system precisely predicted growth and yield of the entire stand from which it was developed and would also do so for similar stands.

Baldwin and Feduccia later joined Zarnoch et al. (1991), to expand on their research to develop growth and yield equations for both thinned and unthinned slash pine, on cutover sites in the West Gulf Coast region. A moment-percentile distribution recovery method using the Weibull distribution was used to estimate diameter distributions. As with loblolly pine equations this study produced equations that estimated variables to within 5% of observed variables.

Matney and Farrar (1992) developed growth and yield equations for planted loblolly pine on thinned and unthinned cutover land in the U.S. Mid-Gulf South that had undergone site preparation. Their equations approximated diameter distributions prior to the first thinning by recovering parameters from a Weibull distribution so its expected and predicted arithmetic and quadratic mean diameters were equal. A tree list was generated from the Weibull distribution at the time of the first thinning and a specified thinning was then applied to this list. A weighted least squares procedure was employed to account for mortality and diameter growth within these tree lists. Matney and Farrar (1992) reported that the least squares adjustment was not developed for biological research; however, it was useful in producing sound tree growth prediction equations. The authors reported that by choosing a weighted function, it was possible to generate tree growth prediction equations that were almost as accurate as equations developed from the regression of the original raw data (Matney and Farrar 1992).
Use of the Weibull function for modeling diameter distributions was first introduced by Bailey and Dell (1973) because it was flexible and produced probabilities without the need for numerical integration (Cao 2004). This function has since been used extensively for this purpose without modification (Baldwin and Feduccia 1987, Zarnoch et al. 1991, Matney and Farrar 1992). Cao (2004) developed and tested two new methods of predicting parameters of the Weibull function for modeling diameters in loblolly pine plantations. These methods used the maximum likelihood estimator and the cumulative distribution function to model diameters. Data were randomly divided into a data set for developing the parameters and a data set for testing the parameters. These two methods both produced better goodness-of-fit statistics than previously used methods (Cao 2004).

PTAEDA is a distance dependent individual tree simulator for old field planted loblolly pine (Daniels and Burkhart 1975). The initial version of the program was developed using a large data set collected from pine plantations on cutover, site-prepared areas. Subsequent versions of PTAEDA have improved the program by increasing its versatility. PTAEDA2 can take into consideration numerous silvicultural treatments available for pine management (Westfall et al. 2004). The computer program has been updated to PTAEDA4.0, and incorporates a stand model for plantation grown loblolly pine on cutover, site-prepared areas developed by using individual trees as the basic modeling unit (Burkhart et al. 2008). The user now has the capability to input the diameter distribution of an existing stand along with the percent of defective trees for each diameter class (Virginia Tech 2011). It also has the ability to be flexible with the selection of trees to be thinned from the stand.
Lenhart (1996) developed versatile and easy to use stand level predictors for loblolly and slash pine plantations in East Texas that predicted yields in cubic foot volumes and green weight in pounds by age, site index, and surviving trees per acre. These predictors were developed using a Schumacher-type function. Lenhart’s (1996) models, however, were not compatible and were developed with data from unmanaged plantations with an average age of 10 years. Coble (2009) later developed new predictors using the MODEL Procedure in SAS/ETS (SAS Institute, Inc. 2004) for the same loblolly and slash pine plantations and compared them to Lenhart’s (1996) earlier predictors. Coble’s models were compatible and used Lenhart’s original data plus an additional 15 years of growth. New models were evaluated with an independent database. They outperformed Lenhart’s models for predicting future yields and basal area per acre for all age classes combined and by five-year age classes (Coble 2009). Coble reported that Lenhart’s models consistently overestimated yields and basal area per acre, but that all models predicted tree survival per acre similarly.

Farrar and Matney (1994) developed growth simulators for thinned and unthinned even-aged natural stands of longleaf pine in the South’s East Gulf region. They used the Weibull-recovery system to estimate stand growth before thinning, and a parameter-free diameter recovery system after thinning. These models were tested against the observed data and performed well (Farrar and Matney, 1994).

Gonzalez-Benecke et al. (2012), developed stand level equations for predicting survival, basal area, dominant height, and yield for longleaf pine plantations in the Western Gulf Coastal Plain before thinning occurred. They also developed equations to predict the stand’s basal area response to thinning. Gonzalez-Benecke et al. validated
their equations using a subset of data from the original data collection used to develop the equations. The authors reported that equations developed in this study for survival, dominant height, and volume all performed within the range of variation of the estimations using other published growth and yield models. The model developed for predicting outside bark volume of longleaf pine plantations under-estimated by about 7% (Gonzalez-Benecke et al. 2012).

Computer programs STEMS and TWIGS are distance independent growth and yield systems originally developed to predict annual diameter growth and mortality of individual trees in the North Central United States (Belcher et al. 1982, Miner et al. 1988) and were later adapted to other regions. Teck and Hilt (1990) developed a model that was incorporated into the TWIGS program to predict survival of individual trees in the Northeastern United States. Their model predicted the survival rate of 19 of 28 species to within 1% of the observed survival rate (Teck and Hilt 1990).

Both STEMS and TWIGS use growth functions to predict potential annual DBH growth along with a modifier to reduce growth due to competition by using different combinations of stand site index, past DBH growth, current DBH, tree height, and crown ratio (Canavan and Ramm 2000). Canavan and Ramm reported an overprediction by these models for DBH growth as well as for mortality.

**Mixed Species Models**

Previous research has primarily been directed toward single species growth and yield due to the simplicity of focusing on one growth habit; however, individual tree growth simulators for multiple species have proven to be flexible tools for predicting forest growth (Pretzsch et al. 2002). The distance-dependent single tree growth
simulator, SILVA, was developed from 155,000 observations, taken from 1952 to 1998, of multiple European species. SILVA’s competition index is a combination of all competitor contributions along with differentiation between deciduous and coniferous species (Pretzsch et al. 2002). SILVA also estimates natural mortality in the stand after calculating the degree of competition. Final output is standard growth and yield information (i.e., stand and stock tables), economic or monetary values, and ecological values which can be used for stand management, research, and education (Pretzsch et al. 2002).

Brooks and Wiant (2004) recognized the need for mixed species growth and yield functions that are easy to use and reasonably accurate. They reviewed three basic but different growth and yield models developed by Spurr (1952) that are simple to apply, but also provide quick and accurate results. Spurr (1952) provided a detailed history of these models that dates to the late 1800s. Two of the models used applied to all stands regardless of species, stocking levels, or age structure and utilized parameters from observed data, basal area per hectare and average stand height. The third model which only utilized basal area per hectare and average stand height along with a stand form factor accounted for 90% of the variation in volume yield in 1952 (Spurr 1952, Brooks and Wiant 2004). Brooks and Wiant (2004) found that the third model, which included stand form factor, actually explained over 99% of the variation in volume yield in pine and hardwood systems and that the addition of an extra model parameter, did not provide a large improvement in prediction accuracy.

Schulte and Buongiorno (2004) presented a growth and yield model for shortleaf pine (*Pinus echinata*) and hardwood stands of all age groups naturally regenerated in the
Southern United States. This model was site and density-dependent and consisted of equations for tree growth, mortality, and recruitment. Predicted ingrowth, mortality, and recruitment rates and tree height and sawlog length from the individual prediction equations all fell within the 95% confidence interval of the observed means with few exceptions (Schulte and Buongiorno 2004). Average diameter class within species group fell within the 95% confidence interval of the average observed number of trees in each group with only a few exceptions. The model presented by Schulte and Buongiorno (2004) predicted that shortleaf pines would eventually be replaced in later years with either hard or soft hardwoods depending on site quality. Because the model gives an average growth prediction, it is best suited for large forested areas with many stands.

**Southern Bottomland Hardwood Models**

According to the 2002 USDA Forest Service Forest Inventory Analysis (Wear and Greis 2002), there were 214 million acres of forest land in the Southern U.S., and of that, 120 million acres consisted of hardwoods. With more than half of the total forest land classified as hardwoods, forest managers need management tools such as growth and yield models on which to base and implement decisions. Southern hardwood stands are usually comprised of a mixture of species making growth and yield modeling much more difficult because of different growth rates and timber quality (McTague et al. 2008). Rauscher et al. (2000) tested the accuracy of 10 growth and yield models developed for the southern Appalachian upland hardwood forests on southern bottomland hardwood forests, and found that the software model SETWIGS (Bolton and Meldahl 1989) was the most accurate. It was able to predict basal area and trees per acre to within plus or minus
15% of the observed values 71% of the time; however, the predictor was biased by under-predicting basal area by 9% and trees per acre by 6% (Rauscher et al. 2000).

McTague et al. (2008) presented a growth and yield model system for stand level and individual trees of hardwood forest types in the Southern U.S. This system was based on 641 permanent plots within nine site types that were randomly placed in stands that varied in stocking, age, structure, and species composition. The stand level system predicted dominant height, survival, basal area prediction and projection, and in-growth component. Individual tree growth and yield predictors were constructed for five of the most common species in the southern hardwood forests, with all other species grouped according to their growth characteristics. Stand level predictors all resulted in an adjusted R² fit indices that ranged from 0.83 to 0.99. Individual tree predictor R² values ranged from 0.91 to 0.99 for future diameter; however, height growth indices were considerably lower ranging from 0.43 to 0.75.

Banzhaf (2009) measured log grades of standing trees in red oak-sweetgum bottomland hardwood forests in Mississippi and Alabama to develop equations that predict merchantable sawtimber volumes and volumes by grade category in trees by species group. Prior to his work, there was essentially no method for estimating the quantity and quality of standing grade hardwood in the Southern United States. Banzhaf (2009) developed two separate sets of equations for each species group that used either total height or merchantable height. Models were chosen based on significance of variables, index of fit (I² is one minus the quantity of the error sum of squares divided by the total sum of squares), root mean squared error (RMSE), bias, ease of use, and biological trends (Banzhaf 2009).
Schultz et al. (2010) analyzed measurements taken in Mississippi and Alabama from 638 stand-level observations on 258 distinct permanent growth and yield plots to develop a red oak-sweetgum growth and yield model for stands existing in the minor stream bottoms of the Mid-South U.S. Measurements taken in 1981, 1988, 1994, and again in 2006 were fitted to equations for predicting average height of dominant and codominant red oaks, and trees per acre, arithmetic mean diameter, quadratic mean diameter, and volume for all species groups. These equations were derived to assist forest landowners and managers with the management of these complex stands by producing expected average yields for natural stands of combined species or species groups (Schultz et al. 2010).

Summary

Measells et al. (2005) surveyed forest landowners in four southern states and reported that 75% were underserved with respect to forestry-related educational programs primarily due their unawareness. In Mississippi alone, 66% of the state is classified as forested of which approximately 3 million acres is bottomland hardwood forests (Matney and Schultz 2011). Nonindustrial private forest landowners own two thirds of this resource. Forest landowners and managers in this region are lacking sufficient growth and yield information to help manage their bottomland hardwoods (Schultz et al. 2008). In particular, a model is needed that can predict future growth and productivity of these valuable forests and thereby enhance their abilities to manage and sustain them. Previous research for red oak-sweetgum bottomland hardwoods by Iles (2008) and Banzhaf (2009) have resulted in equations that predict height and grade but not individual tree DBH or the probability of tree survival. Equations for the prediction of diameter growth and the
probability of individual tree survival based on tree and stand variables were developed based on the datasets utilized by Iles (2008) and Banhzaf (2009).
CHAPTER III

METHODS

Data

A red oak-sweetgum bottomland hardwood growth and yield study was originated in 1981 with funding from the USDA Forest Service Center for Bottomland Hardwoods Research in Stoneville, Mississippi. The initial study consisted of 150 distinct permanent plots primarily of red oak-sweetgum overstory in the minor stream bottoms of Mississippi. Plots were first measured in 1981 and then remeasured in the years of 1988, 1994, and 2006. Measurements and remeasurements collected on plot trees since 1981 create a database of 29,244 tree records. Of these 29,244 trees, 2,103 were professionally graded for the creation of growth and yield models that would predict the volume of merchantable sawtimber by grade category and species group.

Not all of the original 150 plots were still in existence at the time of subsequent remeasurements. In 1988, 144 of the original 150 plots were remeasured, and in 1992 and 1993 only 115 of the original plots were available. At the time of the 1992/1993 remeasurement, 40 new plots were established to replace plots lost to harvesting or natural destruction. In 1994, 31 temporary plots were established to have a sufficient number for development of a sound preliminary stand level growth and yield model. In 2005, only 86 of the original plots were still in existence. Seventy-two additional new
plots were established to replace plots that had either been harvested or could no longer be located bringing the total number of permanent plots for future remeasurement to 158.

Plots were located in even-aged unmanaged stands not disturbed for at least 10 years in Mississippi and Alabama with a minimum of 60 ft²/ac total basal area consisting of at least 30% red oak basal area. Plots ranged in size from 0.1 acre to 1.0 acre. Basal area calculations were based on trees with a DBH greater than or equal to 3.6 inches and in stands at least 20-years-old. There was no maximum age limit. Lands between the Mississippi River and its levee system and those in the loessial hills were excluded from the study since soil origins and native growth capacity are distinctly different from other major and minor stream bottoms.

Measurements were recorded for six species groups; Red Oak, White Oak, Sweetgum, Hickory, Other Commercial, and Non-Commercial, within red oak-sweetgum forest bottomland stands. Cherrybark oak, water oak (*Quercus nigra* L.), and willow oak (*Quercus phellos* L.) were the red oak species that occurred most frequently; while, swamp chestnut oak (*Quercus michauxii* Nutt.), white oak (*Quercus alba* L.), and overcup oak (*Quercus lyrata* Walt.) occurred most frequently for the white oak species. The Other Commercial species group consisted of commercially valuable species that did not occur frequently enough for the development of a species specific growth and yield model. Yellow popular (*Liriodendron tulipifera* L.), green ash (*Fraxinus pennsylvanica* Marsh.), and sugarberry (*Celtis laevigata* Willd.) are examples of other commercial species. The Non-Commercial group consisted of species with no timber value and primarily consisted of American hornbeam (*Carpinus caroliniana* Walt.).

Data collected from all trees greater than 3.6 inches DBH consisted of
1. species,
2. DBH to 0.1 inch,
3. crown class,
4. butt log grade, and
5. azimuth and distance from plot center.

The 1981, 1982, and 1988 measurements included heights on all trees in the plots. In 2007, a representative height sample of 10 trees total from across the range of one-inch DBH classes present on each plot was collected. These height measurements were recorded to the nearest foot and included

1. total,
2. merchantable,
3. 8-inch top,
4. 4-inch top, and
5. base of the live crown.

Tree site index data were collected on six dominant and codominant red oak trees in each stand. Trees located within the plot radius were used for site index measurements when available; however, in the event that plot trees did not qualify for accurate site index determination, trees adjacent to the perimeter of the plot were measured. These measurements included age and total height. Trees on remeasurement plots that had grown into the requirements for study trees or “ingrowth” were added to the plot data. Ingrowth was recorded on each plot by tagging all trees that had grown into the four-inch DBH class. DBH and total height were recorded for each tree, along with the distance and azimuth from plot center to the in-growth tree.
Procedures

Tree data were used to calculate basic tree and stand attributes that were used in the model development process. These attributes were trees per acre (TPA), average diameter growth per year (DG), arithmetic mean diameter (AD) per plot, quadratic mean diameter (QD) per plot, basal area per plot, and cubic foot volume per plot. Average age per plot measured on site index trees was also calculated from ring counts divided by the trees per plot (tpp). Average age for each plot was used instead of individual tree ages. Plot ages equaled the average age of the plot at the last measurement plus the time duration between measurement periods.

Number of trees tallied on each plot was divided by the plot size to calculate TPA, which is the average number of trees present on one acre for the given stand (Equation 1).

\[ TPA = \frac{\sum_{i=1}^{n} tpp_i}{\text{plotsize}} \]  

where

TPA = trees per acre,  
tpp = trees per plot, and  
plotsize = plot area in acres.

Average diameter growth per year was calculated by dividing the difference between DBH measurements by the number of years between last two measurements (Equation 2). This is the average diameter growth of the trees in one year for the individual stand.

\[ DG = \frac{DBH_1 - DBH_0}{Age_1 - Age_0} \]  

(2)
where

DG = average diameter growth per year,
DBH1 = a tree’s diameter at breast height at the beginning of the measurement period,
DBH0 = a tree’s diameter at breast height at the ending of the measurement period,
Age1 = plot age at the beginning of the measurement period, and
Age0 = plot age at the end of the measurement period.

Arithmetic mean diameter (Equation 3) and quadratic mean diameter (Equation 4) were calculated respectively using the following formulas:

\[
AD = \frac{\sum_{i=1}^{n} DBH}{tpp}
\]  
\[
QD = \sqrt{\frac{\sum_{i=1}^{n} DBH^2}{tpp}}
\]

where

AD = arithmetic mean diameter,
QD = quadratic mean diameter,
DBHi = diameter at breast height for an individual tree, and
	tpp = trees per plot.

The standard deviation (s) of DBH (Equation 5) is

\[
s = \sqrt{QD^2 - AD^2} .
\]

Remeasurement data were analyzed to construct individual growth and yield models to estimate diameter distribution volumes based on log and tree grade for red oak-sweetgum forests along with individual tree mortality. Independent variables were the site index equations derived by Iles (2008) and Banzhaf’s (2009) grade equations, age,
dominate height, basal area per acre, and quadratic mean diameter. The primary drivers
to the individual tree model were survival and DBH by species group.

Iles (2008) utilized weighted nonlinear regression to construct a model for
predicting height of dominant and codominant red oaks from the Chapman Richards
segmented model (Matney et al. 1985) (Equation 6):

\[ HD = a \left(1 - e^{b \cdot \text{Age}}\right)^c \]  

where

HD = average height of dominant and codominant red oaks in feet,
Age = average age of dominant and codominant red oaks,
e = base of natural logarithm, and
a, b, c = parameters estimated from the data.

Height of dominant and codominant red oaks is an independent variable for a site
index equation (Iles 2008) which produces an estimate of dominant height, one of the
independent variables used to predict tree growth (Equation 7).

\[ SI = HD \left(\frac{1 - e^{b \cdot I}}{1 - e^{b \cdot \text{Age}}}\right)^c \]  

where

SI = site index (base age 50) of red oaks in feet,
HD = dominant and codominant tree heights in feet,
I = index age of 50 years, and
a, b, c = parameters derived from sample tree measurements.

The final diameter growth equation is not sensitive to site index because there was
not a high level of site index variability among bottomland hardwood sites in the stands
studied. Iles’s site index equation is, however, an adequate estimate of dominant and
codominant height of red oak trees on the site.

Regression techniques were used to determine the level of variation in the
dependent variables associated with the independent variables. Basic regression
assumptions were met for the models to be sufficient. These assumptions were (1) data
were appropriate for the model used, (2) error of variance was constant for all data
observed, (3) errors were independent and random for all variables, (4) no outliers
occurred, (5) errors were normally distributed for all variables, and (6) all important
independent variables were included in the model. Dependent variables were all plotted
against independent variables to determine that these assumptions had been met.

Linear and nonlinear regression procedures were estimated to predict diameter
growth (dependent variable) for each species from tree and stand level attributes
(independent variables). These equations were initially developed from nonlinear models
and then transformed into linear models to obtain initial guesses for nonlinear
estimations. All transformed models were later fitted by nonlinear procedures to avoid
bias associated with predicting variables with transformed data and then inverting the
data back to its original form.

Model performance was judged on root mean squared error (RMSE) for variables
tested. Selection was based on the significance and sensitivity of the variables response
to independent variable changes, visible trends in biological patterns, consistency of the
variance, and finally the fit statistic. Variables tested are listed in Table 1.
Table 1  Dependent and independent variable descriptions tested with regression analysis for the prediction of individual tree diameter growth and survival probability.

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Dependent and Independent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>DBH₀</td>
<td>DBH at the beginning of the measurement period</td>
</tr>
<tr>
<td>DBH₁</td>
<td>DBH at the ending of the measurement period</td>
</tr>
<tr>
<td>DG</td>
<td>(DBH₁ - DBH₀)/(Age₁ - Age₀)</td>
</tr>
<tr>
<td>RDG*</td>
<td>DG*100/DBH₀</td>
</tr>
<tr>
<td>RDG2*</td>
<td>DG*100/quadratic mean diameter</td>
</tr>
<tr>
<td>QD</td>
<td>quadratic mean diameter</td>
</tr>
<tr>
<td>AD</td>
<td>arithmetic mean diameter</td>
</tr>
<tr>
<td>DRat</td>
<td>DBH₀/QD</td>
</tr>
<tr>
<td>Age₀</td>
<td>Stand age at the beginning of the measurement period</td>
</tr>
<tr>
<td>Age₁</td>
<td>Stand age at the ending of the measurement period</td>
</tr>
<tr>
<td>IA or IAge</td>
<td>1/Age₀</td>
</tr>
<tr>
<td>DelAge</td>
<td>Age₁-Age₀</td>
</tr>
<tr>
<td>DRatIAge</td>
<td>(DBH₀/QD)/Age₀</td>
</tr>
<tr>
<td>HD</td>
<td>mean dominant height (Iles, 2008)</td>
</tr>
<tr>
<td>LnHD</td>
<td>Log(HD)</td>
</tr>
<tr>
<td>LnHDIAge</td>
<td>Log(HD)/Age₀</td>
</tr>
<tr>
<td>HDIØD</td>
<td>HD*1/QD</td>
</tr>
<tr>
<td>DIQD</td>
<td>DBH₀/QD</td>
</tr>
<tr>
<td>IADIQD</td>
<td>IA*DIQD</td>
</tr>
<tr>
<td>DIA</td>
<td>DBH₀/Age₀</td>
</tr>
<tr>
<td>TPAAID</td>
<td>TPA/DBH₀</td>
</tr>
<tr>
<td>DIAD</td>
<td>DBH₀/AD</td>
</tr>
<tr>
<td>IADIAD</td>
<td>IA*DIAD</td>
</tr>
<tr>
<td>HDIAD</td>
<td>HD/AD</td>
</tr>
<tr>
<td>IAID</td>
<td>IA/DBH₀</td>
</tr>
<tr>
<td>IsAlive*</td>
<td>Survived till Age₁</td>
</tr>
</tbody>
</table>

*Dependent Variable

Binary logistic regression was used to estimate probabilities of tree survival from the same variables included in the diameter growth model. Only models whose Pearson’s Chi-squared test was significant at the 0.05 level were considered. The best model from this group was selected based on the highest number of concordant pairs and lowest number of discordant pairs.
The individual tree growth and yield system was implemented in Microsoft Visual Basic® editor and a companion Microsoft Visual C++® dynamic link library (dll). The Microsoft Excel® application installer is available to forestry professionals and landowners on the World Wide Web at www.timbercruise.com/Downloads/GYModels/BLHWGYSetup.exe. User interface functionality is provided in the Microsoft Excel® application, and the dll implements the growth and yield system.
CHAPTER IV
RESULTS AND DISCUSSION

Tree Growth Equations

Estimated parameters and fit statistics for the individual tree diameter growth model that best fit all species groups (Equation 8) are given in Table 2.

\[ RDG = a + b \ DBH_0 + c \ IADIQD + d \ TPAID + e \ HD + f \ IAID \]  

(8)

where

RDG = relative diameter growth,
DBH_0 = DBH at the beginning of the measurement period,
IADIQD = inverse of age multiplied by the ratio of DBH_0 to QD,
TPAID = TPA divided by the DBH_0,
HD = average height of dominant and codominant trees in stand (Iles, 2008),
IAID = inverse of age divided by DBH_0, and
a,b,c,d,e,f = parameters to be estimated.
Table 2  Annual percent diameter growth (Equation 8*) parameter estimates, standard error of the estimate (S_{y,x}), and R^2 for red oak-sweetgum stands in the Mid-South U.S.

<table>
<thead>
<tr>
<th>Species Groups</th>
<th>Parameter Estimates</th>
<th>Fit Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>Cherrybark Oak</td>
<td>1.720</td>
<td>-0.0224</td>
</tr>
<tr>
<td>Other Red Oak</td>
<td>2.780</td>
<td>-0.0191</td>
</tr>
<tr>
<td>All Red Oak</td>
<td>2.380</td>
<td>-0.0181</td>
</tr>
<tr>
<td>White Oak</td>
<td>1.660</td>
<td>-0.0241</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>0.529</td>
<td>-0.0161</td>
</tr>
<tr>
<td>Hickory</td>
<td>1.430</td>
<td>-0.0528</td>
</tr>
<tr>
<td>Other Commercial</td>
<td>1.450</td>
<td>-0.0577</td>
</tr>
<tr>
<td>Non-Commercial</td>
<td>1.910</td>
<td>-0.1390</td>
</tr>
</tbody>
</table>

*Equation 8: \( RDG = a + b \text{DBH}_0 + c \text{IADIQD} + d \text{TPAID} + e \text{HD} + f \text{IAID} \)

Table 2 parameter estimates were positive for all species for the ratio of the reciprocal of age to DBH (IAID) (parameter f); and for all species except White Oak for the ratio of the reciprocal of age to DBH divided by quadratic mean diameter (IADIQD) (parameter c). Positive effects of these variables can be explained biologically as younger and smaller trees grow at a faster rate than older larger trees.

DBH at the beginning of the measurement period (DBH_0) has a negative effect (parameter b) on the slope of the regression line. This negative effect can be explained biologically as the greater the tree diameter, the slower the tree grows on a percentage basis. A similar negative effect on slope was also observed from the increase in the ratio of TPA to DBH (parameter d) and average height of the dominant and codominant trees (parameter f). These negative relationships can be explained by the fact that older trees
typically grow in diameter at a slower percentage rate and the greater the competition, the slower the diameter growth.

The fit statistic or $R^2$ for the species groups of White Oak (0.8%), Other Commercial (2.3%), Hickory (3.3%), and Non-Commercial (5.8%) were the lowest reported in Table 2, but exhibited about the same standard error of the estimate ($S_{y,x}$) as species groups with higher $R^2$ values. Having a percent diameter growth constant with low $S_{y,x}$ is ideal as it results in a simpler model with fewer complex interactions. This indicates for the lesser species in the study that their percent diameter growth is uncoupled from the stand condition. This is most probably explained by the Other Commercial and Non-Commercial groups being a combination of species having high shade tolerance. White Oak and Hickory groups are also combinations of different species within the *Quercus* and *Carya* families. These groups may or may not be on sites for optimal growth. Because site, as expressed in the HD variable, is an important component of the model and species in these groups have broader site adaptabilities (elasticity), a lower fit index may be expected. Data showed that independent variables in the equation have less effect on diameter growth of the White Oak species group due to their ability to grow at a fairly constant rate regardless of age, site, or competition.

Separate growth equations were created for Cherrybark Oak (*Quercus pagoda*), all Other Red Oaks except cherrybark oak, and All Red Oak species combined. The diameter growth equation for the Red Oak species group excluding cherrybark oak (Other Red Oak) showed the highest $R^2$ of 34.2% followed by the equation for the Red Oak group including cherrybark (All Red Oaks) (32.6%) and finally the equation for Cherrybark Oak alone (30.4%). A general linear model test for significant differences
among the three red oak models proved negative; therefore, all model fit statistics reported were for the All Red Oak group equation.

The dependent variable is percent annual diameter growth. High $R^2$ values for the Red Oaks and Sweetgum groups indicate these species are sensitive to the independent variables of age, site index, and stand density. Low $R^2$ values for the lesser shade tolerant species groups illustrate, as might be expected, that regardless of stand conditions these species tend to grow at a constant rate.

Percent diameter growth standard error of estimate ($S_{y.x}$) values for all species groups were very low. Values ranged from 0.66% for Sweetgum up to 1.03% for the group containing the other commercial species. This represents the amount of error expected from the equation. These low values are indicative of the suitability of the prediction equations.

Predicted percent diameter growth equations were tested for sensitivity to changes in DBH. Red Oak and Sweetgum species groups showed a positive response in growth as DBH increases probably due to site adaptability of these species groups (Figure 1). Non-Commercial species percent DBH growth declined rapidly with increasing DBH and then flattened out. American hornbeam (*Carpinus caroliniana*), which has a slow growth habit and does not often reach diameters as large as many of the other species, was the primary non-commercial species. White oak, Hickory, and Other Commercial species all showed a gradual negative response to increases in DBH. These species did not occur in large quantities in the red oak-sweetgum mixtures found on the minor stream bottoms of the Mid-South and exhibited lower percent diameter growth than the Red Oak and Sweetgum groups.
Percent growth equations were tested for sensitivity to changes in stand age (Figure 2). Red Oak, Hickory, Other Commercial, and Sweetgum species groups all showed negative responses to increases in stand age. This response is consistent with the slowing growth rate of these groups as the stand ages. White Oak and Non-Commercial groups showed no response in percent growth across stand ages. Predicted White Oak response was probably due to its infrequent occurrence and shade intolerance. The predicted Non-Commercial group response could also be explained by its infrequent occurrence and its slow growth rate.
Figure 2  Predicted percent diameter at breast height (DBH) growth based on changes in age for all species groups.

Percent growth sensitivity to changes in stand TPA is depicted in Figure 3. All species show a negative response to increases in TPA. Red Oak and Sweetgum show a more drastic decline in percent growth compared to the other species groups. This can be attributed to these species lack of tolerance to competition. The Hickory group, which is more tolerant to competition than the other species groups, also declines rapidly in diameter growth. This rapid decline is most probably due to their existence in the codominant, intermediate, and suppressed crown class position during stand development which is characterized by declining percent DBH growth.
Tree Survival Equations

Binary logistic regression was used to develop an equation (Equation 9) for predicting individual tree survival based on independent variables constructed from DBH, age, and stand mean dominate height. Survival model parameter estimates by species group are given in Table 3, and Pearson Chi-squared probabilities (measures of the degree of association between observed and predicted response variables) and logistic regression concordant, discordant, and tie percentages are listed in Table 4.
\[ P = \frac{1}{1 - e^{-(a + b \text{DRat} + c \text{IAge} + d \ln \text{HD} + e \ln \text{HDIAge} + f \text{DRatIAge} + g \text{DelAge})}} \] (9)

where

\( P \) = individual tree survival probability to \( \text{Age}_1 \),

\( \text{DRat} = \frac{\text{DBH}_0}{QD_0} \),

\( \text{IAge} = \frac{1}{\text{Age}_0} \),

\( \ln \text{HD} = \ln \text{base } e \) of the stand’s mean dominant height,

\( \ln \text{HDIAge} = \ln \text{base } e \) of the stand’s mean dominant height divided by \( \text{Age}_0 \),

\( \text{DRatIAge} = \frac{\text{DRat}}{\text{Age}_0} \),

\( \text{DelAge} = \text{Age}_1 - \text{Age}_0 \),

\( a, b, c, d, e, f, g \) = parameters to be estimated from sample tree measurements.

Table 3  
Species group parameter estimates for the model (Equation 9) predicting individual tree survival probability for red oak-sweetgum stands in the Mid-South U.S.

<table>
<thead>
<tr>
<th>Species Groups</th>
<th>Parameter Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a )</td>
</tr>
<tr>
<td>Cherrybark Oak</td>
<td>-10.91</td>
</tr>
<tr>
<td>Other Red Oak</td>
<td>-2.84</td>
</tr>
<tr>
<td>All Red Oak</td>
<td>-1.88</td>
</tr>
<tr>
<td>White Oak</td>
<td>-43.39</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>-18.43</td>
</tr>
<tr>
<td>Hickory</td>
<td>-1.16</td>
</tr>
<tr>
<td>Other Commercial</td>
<td>-2.15</td>
</tr>
<tr>
<td>Non-Commercial</td>
<td>0.92</td>
</tr>
</tbody>
</table>
Table 4 Pearson Chi-squared probabilities and logistic regression concordant-discordant pairs for bottomland hardwood species group equations predicting individual tree survival probability for red oak-sweetgum stands in the Mid-South U.S.

<table>
<thead>
<tr>
<th>Species Groups</th>
<th>Pearson Chi-squared Probabilities</th>
<th>Concordant</th>
<th>Discordant</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cherrybark Oak</td>
<td>0.000</td>
<td>81.6</td>
<td>18.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Other Red Oak</td>
<td>0.000</td>
<td>76.3</td>
<td>23.4</td>
<td>0.3</td>
</tr>
<tr>
<td>All Red Oak</td>
<td>0.000</td>
<td>77.9</td>
<td>21.8</td>
<td>0.3</td>
</tr>
<tr>
<td>White Oak</td>
<td>0.000</td>
<td>68.5</td>
<td>30.7</td>
<td>0.9</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>0.000</td>
<td>76.5</td>
<td>23.1</td>
<td>0.4</td>
</tr>
<tr>
<td>Hickory</td>
<td>0.743</td>
<td>69.2</td>
<td>29.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Other Commercial</td>
<td>0.437</td>
<td>67.5</td>
<td>31.8</td>
<td>0.6</td>
</tr>
<tr>
<td>Non-Commercial</td>
<td>0.103</td>
<td>64.8</td>
<td>34.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Pearson Chi-squared probabilities for all species group equations except Hickory Other Commercial, and Non-Commercial were highly significant (less than 0.01). The null hypothesis that there was no correlation between observed and predicted individual tree survival was rejected for significant species groups.

Logistic regression and concordant and discordant pairs listed in Table 4 indicate how well independent variables predict individual tree survival probabilities. Red oaks were separated into three groups during analysis; however, due to the lack of significant differences between the regressions using the extra sum of squares testing procedure, the All Red Oak equation was chosen for discussion. The All Red Oak species group had the highest percentage of concordant pairs while the Non-Commercial group had the lowest...
percentage. The percentage of concordant pairs for all species groups was 2.1 to 3.5 times greater than the corresponding discordant pairs.

Hickory, Other Commercial, and Non-Commercial species did not occur in great abundance on study plots, and there were insufficient observations to obtain statistically significant models. However, percentages of concordant and discordant pairs together with the sensitivity analyses in Figures 4 and 5 show that the models behave well in application.

Survival probability models were examined for sensitivity to changes in DBH, and results are depicted in Figure 4. All species performed as would be expected from their known biological behavior in bottomlands across the Mid-South. The Red Oak species group exhibited an upward curve for survival indicating that as individual trees increase in size the greater their survival rate increases due to out-growing their competition. Sweetgum survival probability begins to increase as trees reach the 18-inch diameter size class. This, too, is due to growing larger than their competition and living longer than understory species in these stands. Correspondingly, percent survival probability of the non-commercial understory species declined as other species in the stands grew larger and overtook them. Species in the Non-Commercial group are characteristically smaller shade tolerant trees, few of which reach larger diameters. White Oak and Hickory groups were not sensitive to changes in DBH. These species are not typically present on red oak-sweetgum bottomlands, are not broadly-adapted to these sites, and did not occur frequently enough to model adequately.
Sensitivity of predicted survival probability to stand age is depicted in Figure 5. All species groups, with the exception of the Non-Commercial group, show a decline in percent survival as the stand age increases. Sweetgum and White Oak species groups exhibited the most marked response to stand age by declining to zero percent survival at stand age 40. Based on trends in Figures 3 and 4, when a sweetgum lives long enough and achieves main canopy status, its diameter growth and survival chances increase. Non-commercial species showed the only positive trends for stand age. These species are primarily shade tolerant and remain in the understory until light reaches them due to the mortality of an overstory tree.
Computer Program

In addition to stand level component equations, Iles (2008) constructed an individual tree total height equation (Equation 10) for use with the DBH growth and survival equations developed, here, for projecting diameter distributions. He utilized a weighted nonlinear regression model where height of dominant and codominant red oaks, DBH, and quadratic mean diameter (QD) were independent variables:
\[ HT = (a + b HD^c) \left[ 1 - e^{-\left( \frac{DBH}{QD} \right)^g} \right] \]  

(10)

where

HT = individual tree total height in feet,
HD = average height of dominant and codominant trees in stand,
DBH = diameter at breast height in inches,
QD = quadratic mean diameter in inches, and
a,b,c,d,g = parameters to be estimated from data.

This equation along with Banzhaf’s (2009) grade volume equations are implemented in a computer application growth and yield system to predict volume by grade.

The red oak-sweetgum mixture individual tree growth and yield simulator was implemented in Microsoft Visual Basic® Editor within Microsoft Excel® (www.timbercruise.com/Downloads/GYModels/BLHWGYSetup.exe). The growth and yield simulator input screen (Figure 6) allows user input via two methods. Basic stand-level statistics for red oaks may be entered in cells at the top of the input screen. This information consists of age, projected age, height of dominant and codominant red oaks, total TPA, arithmetic mean diameter, quadratic mean diameter, and site index. Alternatively, users may enter a more detailed list of TPA, arithmetic mean diameter, and quadratic mean diameter by species group in the table at the bottom of the screen.

The program’s output screen (Figure 7) consists of two summary tables. The left table is a summary by species group for all DBH and product classes combined. It displays predicted TPA, arithmetic mean diameter, quadratic mean diameter, basal area, and volume by species group. The right table is a stand and stock table that displays predicted height, TPA, basal area, and total volume by species group and DBH class.
The stand and stock table also contains predicted volume by log grade, volume of ties, and cull volume by species and DBH class.

Program outputs may be used by forestry professionals and landowners to project current forest inventories into the future for evaluating and comparing forest management scenarios and making financial decisions.

Figure 6  User input screen of growth and yield simulator for a red oak-sweetgum mixture computer program.
Figure 7 Output screen of growth and yield simulator for a red oak-sweetgum mixture computer program.
**Validation Tests**

Individual tree survival probabilities and percent diameter growth equations were validated on the original data. Validation statistics (i.e., bias, RMSE, index of fit) are given in Tables 5 through 8 for trees per acre, basal area, arithmetic mean diameter, and quadratic mean diameter by individual species groups and for all species combined. Sweetgum and Red Oak species group equations demonstrated relatively low bias, RMSE, and high index of fit statistics across all dependent variables (Tables 5-8). These results indicated higher precision than statistics reported by Rauscher et al. (2000) concerning the SETWIGS program produced by Bolton and Meldahl (1989).

Confidence intervals expressed as a percent on individual stand prediction for the Red Oak and Sweetgum species groups were then calculated from RMSE at the 0.05 level of significance. RMSEs for Red Oaks reported in Tables 5 through 8 are within ±24% for TPA, ±20% for BA, and ±10% for both AD and QD. RMSEs for Sweetgum are within ±30% for TPA, ±22% for BA, ±24% for AD, and ±22% for QD at the 0.05 level of significance. When predictions are applied to multiple stands with the same initial stand attributes, the confidence percents are reduced by a factor of $1/\sqrt{n}$.

White Oak and Other Commercial species groups also showed relatively high bias, RMSE, and low index of fit as compared to the validation statistics for Hickory and Non-Commercial species groups. Percent RMSEs for the prediction of basal area were 71.8% and 87.5% for the Hickory and Non-Commercial species groups, respectively (Table 6). These higher values can be attributed to several factors. Both the Hickory and Non-Commercial groups are combinations of different species that may not occur on every plot and have low plot representation together with a high level of variability.
Table 5  Bias, root mean squared error, and index of fit of estimated trees per acre for all species groups combined and individual species groups for red oak-sweetgum stands in the Mid-South U.S.

<table>
<thead>
<tr>
<th>Species Groups</th>
<th>Bias</th>
<th>RMSE</th>
<th>Index of Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute</td>
<td>%</td>
<td>Absolute</td>
</tr>
<tr>
<td>All Species</td>
<td>-2.35</td>
<td>-1.0</td>
<td>25.59</td>
</tr>
<tr>
<td>Red Oak</td>
<td>0.02</td>
<td>0.0</td>
<td>7.15</td>
</tr>
<tr>
<td>White Oak</td>
<td>-0.03</td>
<td>-0.6</td>
<td>1.96</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>-2.21</td>
<td>-1.6</td>
<td>21.53</td>
</tr>
<tr>
<td>Hickory</td>
<td>-0.01</td>
<td>-0.1</td>
<td>3.12</td>
</tr>
<tr>
<td>Other Commercial</td>
<td>-0.23</td>
<td>-1.3</td>
<td>4.55</td>
</tr>
<tr>
<td>Non-Commercial</td>
<td>0.16</td>
<td>0.9</td>
<td>5.68</td>
</tr>
</tbody>
</table>
Table 6  Bias, root mean squared error, and index of fit of estimated basal area for all species groups combined and individual species groups for red oak-sweetgum stands in the Mid-South U.S.

<table>
<thead>
<tr>
<th>Species Groups</th>
<th>Bias</th>
<th>RMSE</th>
<th>Index of Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute</td>
<td></td>
<td>Absolute</td>
</tr>
<tr>
<td>All Species</td>
<td>1.51</td>
<td>1.1</td>
<td>10.27</td>
</tr>
<tr>
<td>Red Oak</td>
<td>1.38</td>
<td>1.9</td>
<td>7.33</td>
</tr>
<tr>
<td>White Oak</td>
<td>-0.11</td>
<td>-3.5</td>
<td>1.62</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>-0.11</td>
<td>-0.2</td>
<td>5.70</td>
</tr>
<tr>
<td>Hickory</td>
<td>0.06</td>
<td>1.7</td>
<td>2.64</td>
</tr>
<tr>
<td>Other Commercial</td>
<td>0.03</td>
<td>0.5</td>
<td>2.78</td>
</tr>
<tr>
<td>Non-Commercial</td>
<td>0.27</td>
<td>7.6</td>
<td>3.07</td>
</tr>
</tbody>
</table>

Table 7  Bias, root mean squared error, and index of fit of estimated arithmetic mean diameter for all species groups combined and individual species groups for red oak-sweetgum stands in the Mid-South U.S.

<table>
<thead>
<tr>
<th>Species Groups</th>
<th>Bias</th>
<th>RMSE</th>
<th>Index of Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute</td>
<td></td>
<td>Absolute</td>
</tr>
<tr>
<td>All Species</td>
<td>0.11</td>
<td>1.2</td>
<td>0.43</td>
</tr>
<tr>
<td>Red Oak</td>
<td>0.17</td>
<td>1.1</td>
<td>0.86</td>
</tr>
<tr>
<td>White Oak</td>
<td>-0.16</td>
<td>-3.8</td>
<td>1.26</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>0.03</td>
<td>0.4</td>
<td>1.09</td>
</tr>
<tr>
<td>Hickory</td>
<td>-0.11</td>
<td>-3.1</td>
<td>1.40</td>
</tr>
<tr>
<td>Other Commercial</td>
<td>-0.21</td>
<td>-3.3</td>
<td>1.77</td>
</tr>
<tr>
<td>Non-Commercial</td>
<td>-0.24</td>
<td>-5.6</td>
<td>1.70</td>
</tr>
</tbody>
</table>
Table 8  Bias, root mean squared error, and index of fit of estimated quadratic mean diameter for all species groups combined and individual species groups for red oak-sweetgum stands in the Mid-South U.S.

<table>
<thead>
<tr>
<th>Species Groups</th>
<th>Bias</th>
<th>RMSE</th>
<th>Index of Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute</td>
<td>%</td>
<td>Absolute</td>
</tr>
<tr>
<td>All Species</td>
<td>0.11</td>
<td>1.0</td>
<td>0.47</td>
</tr>
<tr>
<td>Red Oak</td>
<td>0.13</td>
<td>0.8</td>
<td>0.79</td>
</tr>
<tr>
<td>White Oak</td>
<td>-0.19</td>
<td>-4.4</td>
<td>1.35</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>0.02</td>
<td>0.3</td>
<td>1.10</td>
</tr>
<tr>
<td>Hickory</td>
<td>-0.13</td>
<td>-3.3</td>
<td>1.58</td>
</tr>
<tr>
<td>Other Commercial</td>
<td>-0.24</td>
<td>-3.5</td>
<td>1.80</td>
</tr>
<tr>
<td>Non-Commercial</td>
<td>-0.24</td>
<td>-5.4</td>
<td>1.77</td>
</tr>
</tbody>
</table>

Figures 8 through 10 depict TPA residual errors (diff) versus BA at Age₀, Age₀, and QD at Age₀. Stand Age₀ is the stand age at the beginning of the Red Oak species group growth period. Figures 11 through 13 show BA residual errors versus Age₀, QD at Age₀, and TPA at Age₀. Figures 14 through 16 show the AD residual errors versus Age₀, BA at Age₀, and TPA at Age₀. Figures 17 through 19 depict the QD residual errors versus Age₀, BA at Age₀, and TPA at Age₀. None of the plots showed any decided trends about the zero line for any of the independent variables.
Figure 8  Associated trees per acre residual errors (TPA diff) for the Red Oak species groups by plot basal area (BA) at Age₀ (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.

Figure 9  Associated trees per acre residual errors (TPA diff) for Red Oak species group by plot age at the beginning of the growth period (Age₀) for red oak-sweetgum stands in the Mid-South U.S.
Figure 10  Associated trees per acre residual errors (TPA diff) for Red Oak species group by quadratic mean diameter (QD) at Age₀ (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.

Figure 11  Associated basal area residual errors (BA diff) for Red Oak species group by plot age at the beginning of the growth period (Age₀) for red oak-sweetgum stands in the Mid-South U.S.
Figure 12  Associated basal area residual errors (BA diff) for Red Oak species group by quadratic mean diameter (QD) at Age₀ (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.

Figure 13  Associated basal area residual errors (BA diff) for Red Oak species group by trees per acre (TPA) at Age₀ (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.
Figure 14  Associated arithmetic mean diameter residual errors (AD diff) for Red Oak species group by plot age at the beginning of the growth period (Age₀) for red oak-sweetgum stands in the Mid-South U.S.

Figure 15  Associated arithmetic mean diameter residual errors (AD diff) for Red Oak species group by plot basal area (BA) at Age₀ (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.
Figure 16  Associated arithmetic mean diameter residual errors (AD diff) for Red Oak species group by trees per acre (TPA) at Age$_0$ (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.

Figure 17  Associated quadratic mean diameter residual errors (QD diff) for Red Oak species group by plot age at the beginning of the growth period (Age$_0$) for red oak-sweetgum stands in the Mid-South U.S.
Figure 18  Associated quadratic mean diameter residual errors (QD diff) for Red Oak species group by plot basal area (BA) at Age0 (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.

Figure 19  Associated quadratic mean diameter residual errors (QD diff) for Red Oak species group by trees per acre (TPA) at Age0 (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.
Figures 20 through 31 depict Sweetgum TPA, BA, AD, and QD residual errors versus the independent variables (TPA at Age₀, Age₀, and BA at Age₀) of the regressions. None of the plots showed any decided trends about the zero line for any of the independent variables.

Figure 20  Associated trees per acre residual errors (TPA diff) for Sweetgum by plot basal area (BA) at Age₀ (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.
Figure 21  Associated trees per acre residual errors (TPA diff) for Sweetgum by plot age at the beginning of the growth period (Age0) for red oak-sweetgum stands in the Mid-South U.S.

Figure 22  Associated trees per acre residual errors (TPA diff) for Sweetgum by quadratic mean diameter (QD) at Age0 (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.
Figure 23  Associated basal area residual errors (BA diff) for Sweetgum by plot age at the beginning of the growth period (Age0) for red oak-sweetgum stands in the Mid-South U.S.

Figure 24  Associated basal area residual errors (BA diff) for Sweetgum by quadratic mean diameter (QD) at Age0 (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.
Figure 25  Associated basal area residual errors (BA diff) for Sweetgum by trees per acre (TPA) at Age$_0$ (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.

Figure 26  Associated arithmetic mean diameter residual errors (AD diff) for Sweetgum by plot age at the beginning of the growth period (Age$_0$) for red oak-sweetgum stands in the Mid-South U.S.
Figure 27  Associated arithmetic mean diameter residual errors (AD diff) for Sweetgum group by plot basal area (BA) at Age₀ (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.

Figure 28  Associated arithmetic mean diameter residual errors (AD diff) for Sweetgum group by trees per acre (TPA) at Age₀ (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.
Figure 29  Associated quadratic mean diameter residual errors (QD diff) for Sweetgum group by plot age at the beginning of the growth period (Age0) for red oak-sweetgum stands in the Mid-South U.S.

Figure 30  Associated quadratic mean diameter residual errors (QD diff) for Sweetgum by plot basal area (BA) at Age0 (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.
Figure 31  Associated quadratic mean diameter residual errors (QD diff) for Sweetgum by trees per acre (TPA) at Age0 (stand age at beginning of growth period) for red oak-sweetgum stands in the Mid-South U.S.

Figures 32 through 47 illustrate the comparison of observed and predicted diameter distribution values for the Red Oak and Sweetgum species groups by average size class. These figures show that the predicted TPA by DBH class is in close correspondence to the observed. The closeness of the observed and predicted values in addition to the low RMSE and high index of fit for the commercially important species groups further validates that model precision is sufficient to provide good expected yields. Observed and predicted TPA by DBH class comparisons for species groups of lesser commercial value are given in Figures 48 through 73 in Appendix A.
Figure 32  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Red Oak species group with all size classes combined.

Figure 33  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Red Oak species group with an average size class of 8 inches.
Figure 34  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Red Oak species group with an average size class of 9 inches.

Figure 35  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Red Oak species group with an average size class of 10 inches.
Figure 36  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Red Oak species group with an average size class of 11 inches.

Figure 37  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Red Oak species group with an average size class of 12 inches.
Figure 38  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Red Oak species group with an average size class of 13 inches.

Figure 39  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Red Oak species group with an average size class of 14 inches.
Figure 40  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for Sweetgum with all size classes combined.

Figure 41  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for Sweetgum with an average size class of 8 inches.
Figure 42  
Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for Sweetgum with an average size class of 9 inches.

Figure 43  
Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for Sweetgum with an average size class of 10 inches.
Figure 44  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for Sweetgum with an average size class of 11 inches.

Figure 45  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for Sweetgum with an average size class of 12 inches.
Figure 46  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for Sweetgum with an average size class of 13 inches.

Figure 47  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for Sweetgum with an average size class of 14 inches.
In summary, statistical and graphical results showed that the sets of equations predicting individual tree percent diameter growth and survival were biologically consistent and had the highest precision of all the models considered. No other combination of individual variables was found to contribute significantly to improving the models.
CHAPTER V
CONCLUSIONS

Red oak-sweetgum stands on Mid-South United States minor stream bottoms are a very productive and valuable natural resource. Forestry professionals and landowners have been unable to predict future growth and survival, and, therefore, lacked information on which to base management decisions. To provide a growth and yield projection tool for this important forest mixture, individual tree percent annual diameter growth and survival probability models were constructed and fitted to the data using linear, non-linear, and logistic regression techniques on data collected over a 35-year time period. The best set of developed equations was selected based on joint behavior when inserted into the model and the performance of each plot’s predicted future yield when compared to its observed data at the next projection period. Selected equations were incorporated into a growth and yield system implemented in Microsoft Visual Basic® Editor within Microsoft Excel®.

R-squared ($R^2$) values for the parameter estimates of the individual tree annual percent diameter growth model ranged from 0.8% to 30.4%, with the highest values associated with the more commercially important Red Oak group. Percent diameter growth standard error of estimates ($S_{y,x}$) ranged from 0.66% to 1.03%, the lowest of which occurred for the Red Oak and Sweetgum groups. For all species excluding the
Red Oak and Sweetgum groups, the low variation of the growth and survival percentages and the lack of relationship of these species to stand characteristics made models simple, as all of the low frequency species are growing at about the same rate across age, stand density, and site.

Individual tree binary logistic survival equations produced parameter estimates with highly significant Pearson Chi-squared test results for Red Oak and Sweetgum groups. Concordant pairs were 2.1 to 3.5 times more frequent than corresponding discordant pairs suggesting that the models were well-behaved in application.

The Microsoft Visual Basic® Editor growth and yield simulator was validated with the original data for the Red Oak and Sweetgum species groups. TPA, basal area, arithmetic mean diameter, and quadratic mean diameter were compared against the original data for bias, RMSE, and index of fit. For all variables, Red Oak and Sweetgum groups had low bias and RMSE and high R² values, demonstrating higher precision than reported by other models (Rauscher et al. 2000). Statistical and graphical validation techniques showed that the set of individual tree percent diameter growth and survival equations were biologically consistent and provided the highest precision of all models considered.

The resulting Microsoft Excel® growth and yield simulator and user manual can be downloaded free of charge from www.timbercruise.com/Downloads/GYModels/BLHWGYSetup.exe. This application enables forestry professionals and landowners to make better management decisions for their red oak-sweetgum mixture bottomland hardwood stands by projecting current forest
inventories into the future, predicting average yields, and evaluating and comparing forest management scenarios.


Burkhart, H.E., Amateis, R., Westfall, J., and Daniels, R. 2008. PTAEDA4.0: simulation of individual tree growth, stand development and economic evaluation in loblolly pine plantations. Department of Forestry, Virginia Polytechnic Institute and State University, Blacksburg, VA.


APPENDIX A

COMPARISON OF OBSERVED AND PREDICTED DIAMETER DISTRIBUTIONS

WHITE OAK, HICKORY, OTHER COMMERCIAL AND NON-COMMERCIAL SPECIES GROUPS BY SIZE CLASS
Figure 48  
Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for all species groups and size classes combined.

Figure 49  
Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for all species groups combined with an average diameter class of 8 inches.
Figure 50  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for all species groups combined with an average diameter class of 9 inches.

Figure 51  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for all species groups combined with an average diameter class of 10 inches.
Figure 52  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for all species groups combined with an average diameter class of 11 inches.

Figure 53  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for all species groups combined with an average diameter class of 12 inches.
Figure 54  
Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for all species groups combined with an average diameter class of 13 inches.

Figure 55  
Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for all species groups combined with an average diameter class of 14 inches.
Figure 56  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the White Oak species group with all size classes combined.

Figure 57  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the White Oak species group with an average size class of 8 inches.
Figure 58   Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the White Oak species group with an average size class of 9 inches.

Figure 59   Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the White Oak species group with an average size class of 10 inches.
Figure 60  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the White Oak species group with an average size class of 11 inches.

Figure 61  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the White Oak species group with an average size class of 12 inches.
Figure 62  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the White Oak species group with an average size class of 13 inches.

Figure 63  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the White Oak species group with an average size class of 14 inches.
Figure 64  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Hickory species group with all size classes combined.

Figure 65  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Hickory species group with an average size class of 8 inches.
Figure 66  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Hickory species group with an average size class of 9 inches.

Figure 67  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Hickory species group with an average size class of 10 inches.
Figure 68  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Hickory species group with an average size class of 11 inches.

Figure 69  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Hickory species group with an average size class of 12 inches.
Figure 70  
Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Hickory species group with an average size class of 13 inches.

Figure 71  
Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Hickory species group with an average size class of 14 inches.
Figure 72  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Other Commercial species group with all size classes combined.

Figure 73  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Other Commercial species group with an average size class of 8 inches.
Figure 74  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Other Commercial species group with an average size class of 9 inches.

Figure 75  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Other Commercial species group with an average size class of 10 inches.
Figure 76  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Other Commercial species group with an average size class of 11 inches.

Figure 77  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Other Commercial species group with an average size class of 12 inches.
Figure 78  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Other Commercial species group with an average size class of 13 inches.

Figure 79  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Other Commercial species group with an average size class of 14 inches.
Figure 80  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Non-Commercial species group with all size classes combined.

Figure 81  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Non-Commercial species group with an average size class of 8 inches.
Figure 82  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Non-Commercial species group with an average size class of 9 inches.

Figure 83  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Non-Commercial species group with an average size class of 10 inches.
Figure 84  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Non-Commercial species group with an average size class of 11 inches.

Figure 85  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Non-Commercial species group with an average size class of 12 inches.
Figure 86  Comparison of observed and predicted trees per acre (TPA) by diameter at breast height (DBH) class in inches for the Non-Commercial species group with an average size class of 14 inches.